Viscoelastic Characterization of Medical Tape Using the Standard Linear Solid Model

Course: Mechanobiology - ME 618

Instructor: Dr.Farid Alisafaei

Name: Zinia Sultana Joti

NJIT ID: 31730019

1 Introduction

Viscoelastic materials represents both elastic and viscous mechanical behavior. These materials deformation and stress response depend on time, whereas, elastic materials can deform and recover instantly after unloading. Several biological tissues exhibit viscoelastic behavior. These materials exhibit rate-, time- and amplitude-dependent characteristics.

In this project, we investigated the viscoelastic behavior of a medical tape material using creep test and stress relaxation test. For creep test, a constant stress σ_0 is aplied and the resulting strain evolution is measured over time. In a stress relaxation test, a constant strain ϵ_0 is applied and the stress required to maintain the given strain is monitored over time. The stress gradually decreases inside viscous materials.

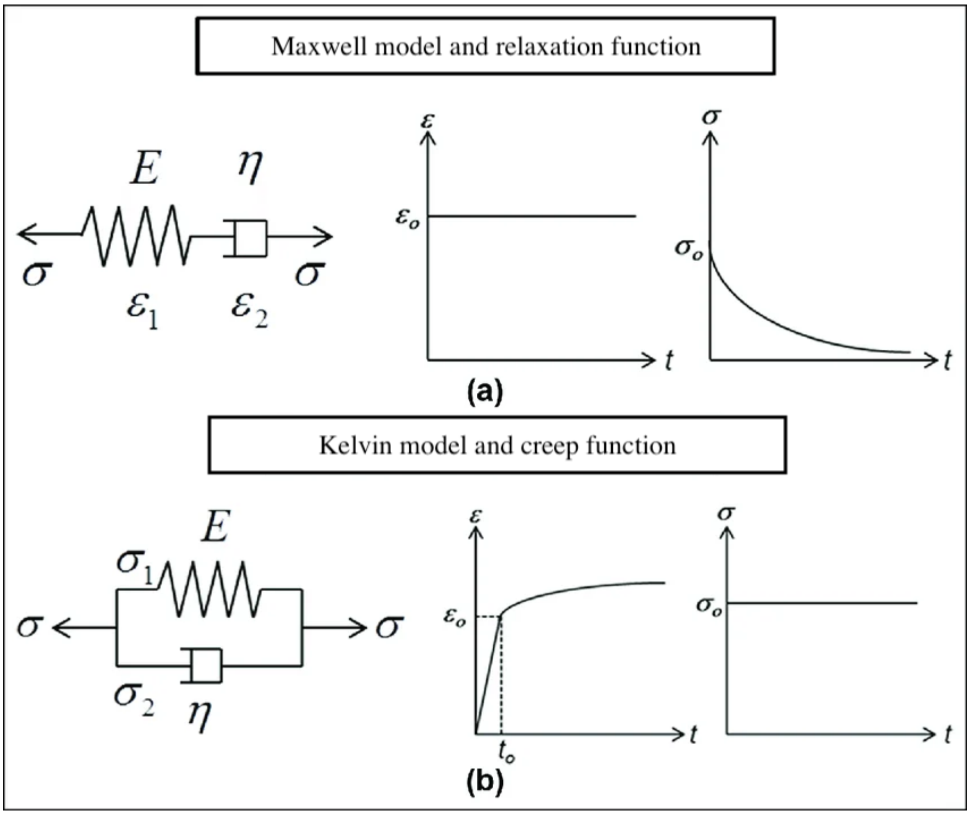

Figure 1: One-dimensional viscoelastic models: (a) Maxwell fluid model; (b) Kelvin solid model

There are many constitutive models developed to describe viscoelastic behavior. The Maxwell model,which consists of a spring and a dashpot in series, captures stress-relaxation behavior effectively but fails to represent creep behavior. On the other hand, Kelvin model, consisting a spring and dashpot in parallel, captres the creep behavior well but cannot accurately estimate the stress-relaxation. A simple one dimensional representation of the models are shown in Figure 1. More complex models can be constructed using more elements.

A complex viscoelastic-reologicacl model can be formed by combining a Maxwell model with an additional spring in parallel. In this way, the model can capture both creep and stress relaxation behavior.This model is known as Standard Linear Solid (SLS) model. Due to its simplicity and ability to capture both time-dependent deformation and stress decay, SLS was chosen for this project. Experimental data were fitted using nonlinear least-squaares optimization in Python to determine the material's parameters for this model.

2 Standard Linear Solid Model

The Standard Linear Solid (SLS) model consists of a Maxwell element connected in parallel with an additional elastic spring. The Maxwell element itself contains a spring and a dashpot connected in series. The model parameters are:

- E_1: Elastic modulus of the parallel spring
- E_2: Elastic modulus of the Maxwell spring
- η: Viscosity coefficient of the dashpot

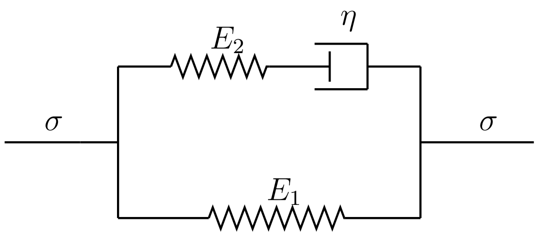

Figure 2: Standard Linear Solid model

3 Derivation of the Governing Equation

Let the total stress and strain in the system be denoted by σ and ϵ, respectively.
Because the Maxwell branch and the spring E_1 are connected in parallel, the strain in both branches is identical:

$$\epsilon = \epsilon_1 = \epsilon_M$$

The stress carried by the parallel spring is given by Hooke's law:

$$\sigma_1 = E_1\epsilon$$

The total stress is the sum of the stresses carried by the two parallel branches:

$$\sigma = \sigma_1 + \sigma_M$$

Substituting the stress in the elastic spring:

$$\sigma = E_1\epsilon + \sigma_M$$

Therefore:

$$\sigma_M = \sigma - E_1\epsilon$$

For the Maxwell element, the constitutive relation is:

$$\dot{\epsilon} = \frac{\dot{\sigma}_M}{E_2} + \frac{\sigma_M}{\eta}$$

Substituting:

$$\sigma_M = \sigma - E_1\epsilon$$

gives:

$$\dot{\sigma}_M = \dot{\sigma} - E_1\dot{\epsilon}$$

Therefore:

$$\dot{\epsilon} = \frac{\dot{\sigma} - E_1\dot{\epsilon}}{E_2} + \frac{\sigma - E_1\epsilon}{\eta}$$

Multiplying both sides by E_2:

$$E_2\dot{\epsilon} = \dot{\sigma} - E_1\dot{\epsilon} + \frac{E_2}{\eta}\sigma - \frac{E_1E_2}{\eta}\epsilon$$

Rearranging terms:

$$\dot{\sigma} + \frac{E_2}{\eta}\sigma = (E_1 + E_2)\dot{\epsilon} + \frac{E_1E_2}{\eta}\epsilon$$

Multiplying the entire equation by $\frac{\eta}{E_2}$ yields the final constitutive equation for the Standard Linear Solid model:

$$\boxed{\sigma + \frac{\eta}{E_2}\dot{\sigma} = E_1\epsilon + \eta\left(1 + \frac{E_1}{E_2}\right)\dot{\epsilon}} \tag{1}$$

3.1 Creep Response

For the creep test, a constant stress σ_0 is applied:

$$\sigma(t) = \sigma_0$$

and therefore:

$$\dot{\sigma}(t) = 0$$

Substituting into the constitutive equation:

$$\frac{E_1}{\eta}\epsilon + \left(1 + \frac{E_1}{E_2}\right)\dot{\epsilon} = \frac{\sigma_0}{\eta} \tag{2}$$

We can solve the differential equation with the initial condition

$$\epsilon(0) = \frac{\sigma_0}{E_1 + E_2}$$

the solution of the creep response using Wolfram:

$$\boxed{\epsilon(t) = \frac{\sigma_0}{E_1}\left[1 - \frac{E_2}{E_1+E_2}\exp\left(-\frac{E_1E_2}{\eta(E_1+E_2)}t\right)\right]} \tag{3}$$

3.2 Stress Relaxation Response

For stress relaxation, a constant strain $\epsilon(t) = \epsilon_0$ is applied. Thus:

$$\dot{\epsilon}(t) = 0$$

Substituting into the constitutive equation:

$$\frac{\sigma}{\eta} + \frac{\dot{\sigma}}{E_2} = \frac{E_1\epsilon_0}{\eta} \tag{4}$$

Solving the equation with the initial condition on Wolfram

$$\sigma(0) = (E_1 + E_2)\epsilon_0$$

gives:

$$\boxed{\sigma(t) = \epsilon_0\left[E_1 + E_2\exp\left(-\frac{E_2}{\eta}t\right)\right]} \tag{5}$$

4 Experimental Setup

The medical tape was prepared in a dogbone-shaped geometry. The specimen dimensions are illustrated in Figure 3, width: 5.56 mm nd thickness: 1.9 mm.

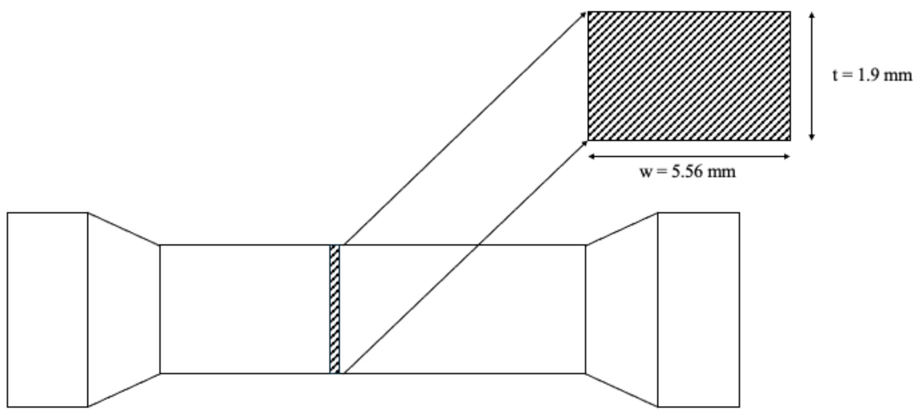

Figure 3: Test specimen in dogbone shape

The specimen was mounted in the testing apparatus shown in Figure 4, and ink markers were applied to facilitate image-based strain tracking during deformation (Figure 4b). The cross-sectional area of the sample was calculated as:

$$A = (5.56 \times 10^{-3})(1.9 \times 10^{-3}) = 1.0564 \times 10^{-5}\ \mathrm{m}^2$$

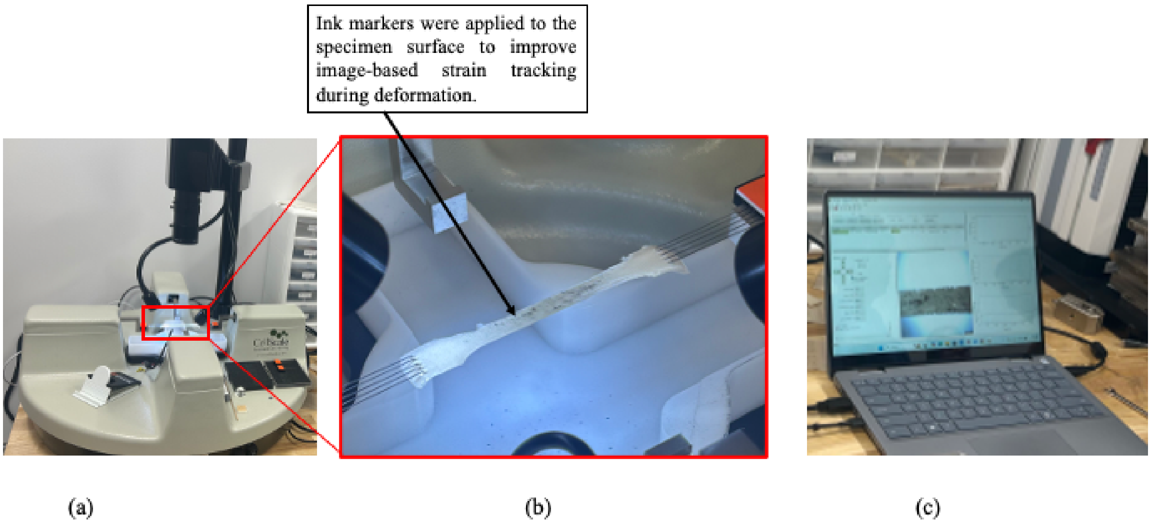

Figure 4: Experimental setup (a) Mechanical testing apparatus used during creep and stress relaxation experiments. (b) Dogbone-shaped medical tape specimen (c) Real-time experimental data acquisition and image-processing interface.

Force measurements obtained experimentally were converted into engineering stress using:

$$\sigma = \frac{F}{A}$$

The experimental data were processed using Python. Nonlinear least-squares optimization was performed using the `curve_fit` function from the SciPy library in order to determine the SLS model parameters.

5 Results

The Standard Linear Solid model was fitted separately to the creep and stress relaxation experimental datasets.

Figure 5 shows the experimental creep response together with the fitted SLS model prediction. The strain increased gradually over time under approximately constant stress loading.

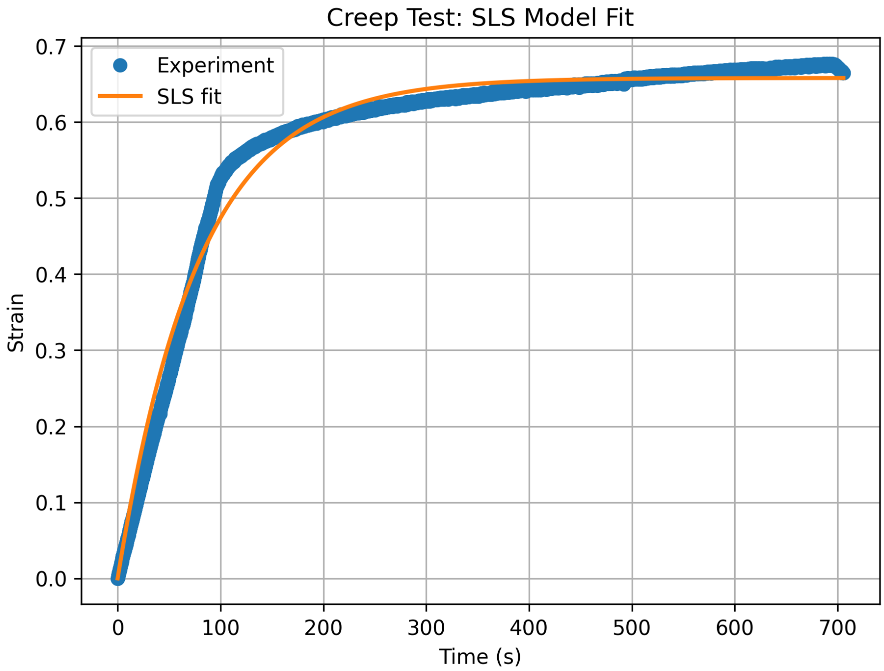

Figure 5: Experimental creep response and fitted Standard Linear Solid model prediction.

Figure 6 shows the stress relaxation response of the material together with the fitted SLS model prediction. The stress decayed gradually over time under approximately constant strain conditions.

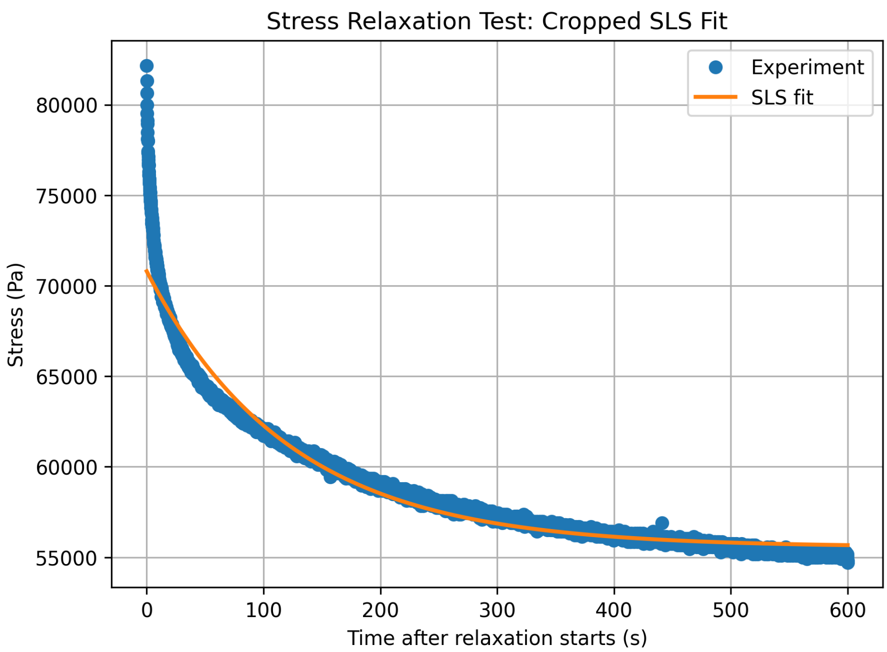

Figure 6: Experimental stress relaxation response and fitted Standard Linear Solid model prediction.

The fitted SLS material parameters obtained from the creep and stress relaxation tests are summarized in Table 1.

Table 1: Fitted Standard Linear Solid model parameters.

Test	E_1 (Pa)	E_2 (Pa)	η (Pa·s)
Creep Test	140843.6051652542	2240366492569.232	11058970.51686586
Stress Relaxation Test	165389.2819441156	45364.86957904435	5538772.869627094
Average	153116.4435546849	1120183268967.0508	8298871.693246477

6 Discussion

The creep test showed that strain increased with time under approximately constant loading (Figure 5), while the stress decreases rapidly under approximately constant strain (Figure 6).
The experimental results were consistent with viscoelastic material behavior. Table 1 summerized the SLS model parameters obtained from the creep and stress relaxation test. The creep test yielded relatively larger values for E_2 and η indicating stronger resistance during constant loading. On the contrary, the stress relaxatoin test produced comparatively lower values since stress decay behavior were observed experimentally.
The fitted parameter obtained from two experiments showed discrepancies. There may have several factors to consider for this difference. One reason can be due to lack of multiple experimental datasets. Only a single experimental dataset was available for each test, which prevents statistical averaging and uncertainty analysis
Nevertheless, the model successfully reproduced the primary viscoelastic behavior of the medical tape and provided a reasonable approximation of its mechanical response.

7 Conclusion

This project was implemented with the objective of learning how to determine the characteristics of a new material. In biomedical field, researchers come across many challenges on how to integrate a novel material with biological cell, tissues etc. Before applying the material, it is necessary to determine the characteristics behavior.
In this study, the viscoelastic behavior of a medical tape specimen was characterized using creep and stress relaxation experiments together with the Standard Linear Solid model. The constitutive equation was derived analytically, and the parameters of the model were determined using nonlinear least-squares fitting in Python. The fitted SLS model captured the primary viscoelastic trends effectively, which was consistent with the experimental data.
Overall, the Standard Linear Solid model provided a simple framework for estimating the mechanical properties of the medical tape.

8 Supporting materials

The code for this project is available at
`https://github.com/ZiniaJoti/sls-viscoelastic-model-fitting`

Acknowledgement

I would like to sincerely thank Professor Farid Alisafaei for his guidance and support throughout the Mechanobiology course for Spring 2026 semester. The laboratory experiments and discussions provided important practical insight into viscoelastic material characterization and constitutive modeling.

Additionally, I gratefully acknowledge the support of the MechanoBiology and BioMechanics (MBBM) Lab for providing the experimental setup, laboratory facilities, and testing resources used in this project.